Switched Hawkes Processes

Namrata Nadagouda and Mark A. Davenport

Georgia Institute of Technology

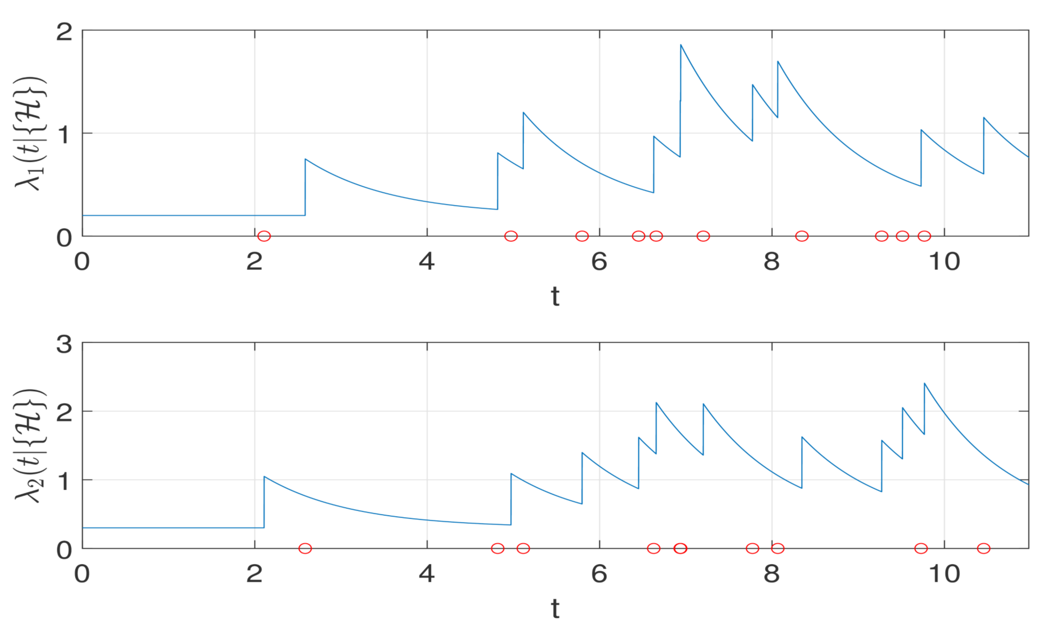
Background

Hawkes processes are similar to Poisson processes, but with a dependence on the process history.

$$\lambda(t|\{\mathcal{H}\}) = \mu(t) + A\sum_{k=1}^{K}\gamma(t-t_k)$$

Multidimensional Hawkes processes can model excitations in a network.

$$\lambda_i(t|\{\mathcal{H}\}) = \mu_i(t) + \sum_{j=1}^{N} A_{ij}\sum_{k\in K_j}\gamma(t-t_k)$$



Used to model data in social networks, finance, geophysical phenomena

Maximum Likelihood Parameter Estimation

$$\hat{x} = \underset{\mu,A}{\text{arg min}}\ \mathcal{L}(\mu, A|\{\mathcal{H}\})$$

$$\mathcal{L}(\mu, A|\{\mathcal{H}\}) = \sum_{i=1}^{N}\sum_{s=1}^{M}\mathcal{L}_{i,s}(\mu, A|\{\mathcal{H}\})$$

$\lambda_{i,s}(t)$ depends only on $\mu_{i,s}$ and a single row of A

MN independent subproblems optimizing over (MN + 1) parameters

$$\mathcal{L}_{i,s}(\mu, A|\{\mathcal{H}\}) = \int_0^T \lambda_{i,s}(t)dt - \sum_{k\in K_{i,s}}\log\lambda_{i,s}(t_k)$$

$$\hat{x}_{i,s} = \underset{\mu_{i,s},A_{i,s}}{\text{arg min}}\ \mathcal{L}_{i,s}(\mu, A|\{\mathcal{H}\})$$

$\mathcal{H}$ - process history

μ_i, $\mu_{i,s}$ - base intensity of the i^{th} process and during s^{th} state of the system

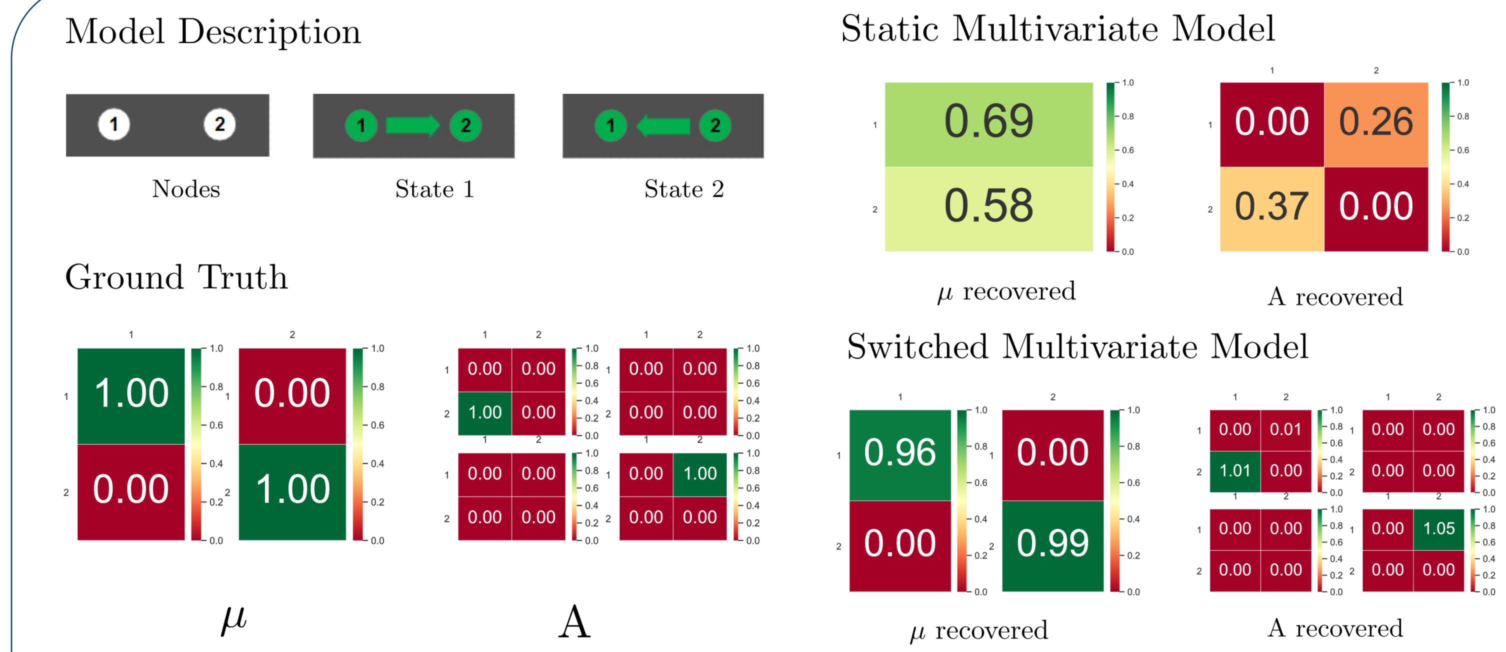
N/M - number of processes/states in the system

A_{ij} - quantifies the influence of the j^{th} process on the i^{th} process

$(A_{ij})_{s,s'}$ - quantifies the influence of events of process j occured during s'^{th} state of the system, on the process i during s^{th} state of the system

K_j, $K_{j,s'}$ - set of events on the j^{th} process and during s'^{th} state of the system

Synthetic Simulations

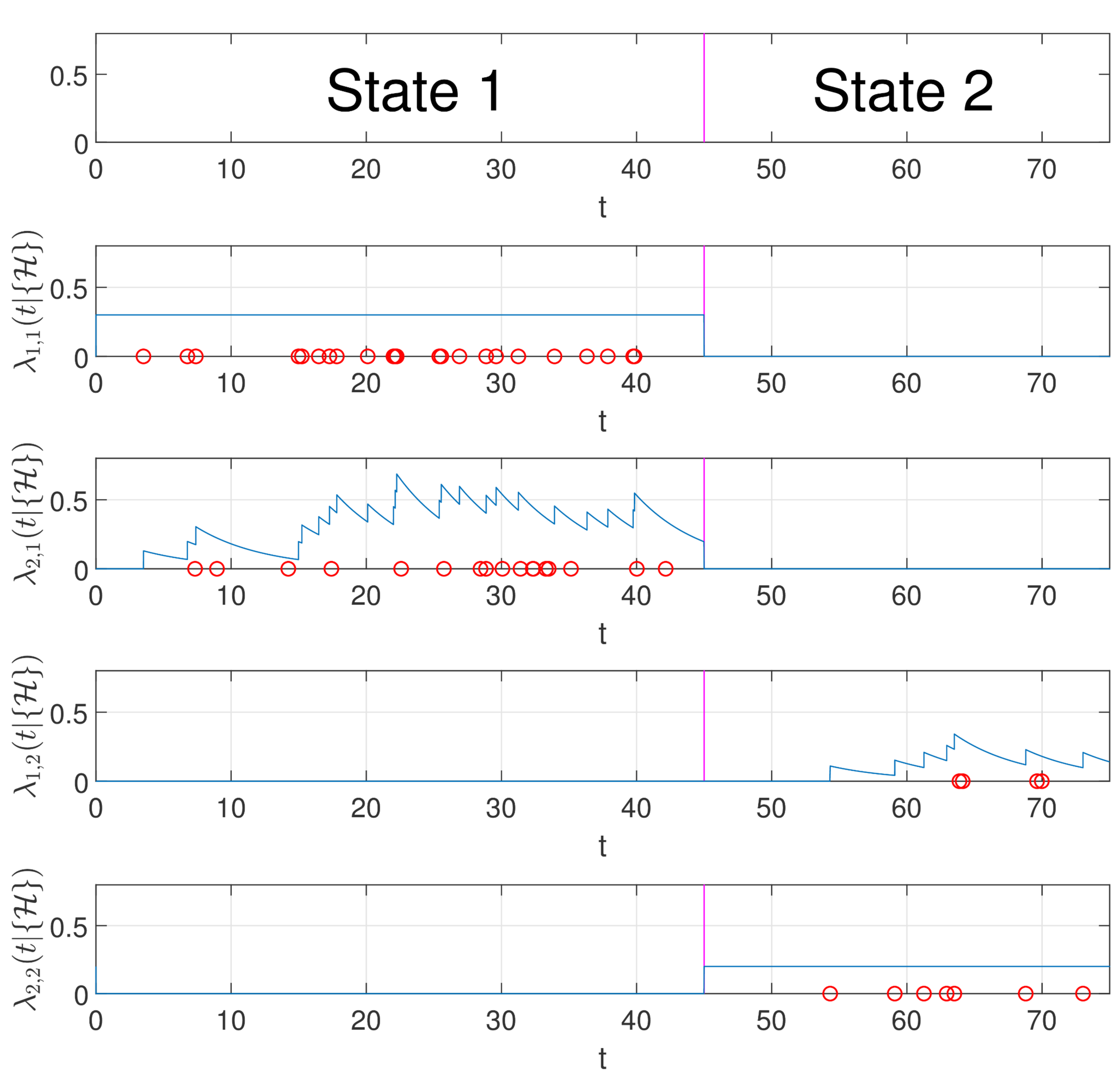


- State 1 - Innovation events at node 1. Excitations at node 2.
- State 2 - Innovation events at node 2. Excitations at node 1.
- The static model learns the avearge behaviour of the system over the two states. The switched model more accurately picks up the base and excitation intensities corresponding to the underlying state of the system.
- The parameters learnt by the switched make more sense in the real world scenario making it better interpretable.

Switched Model

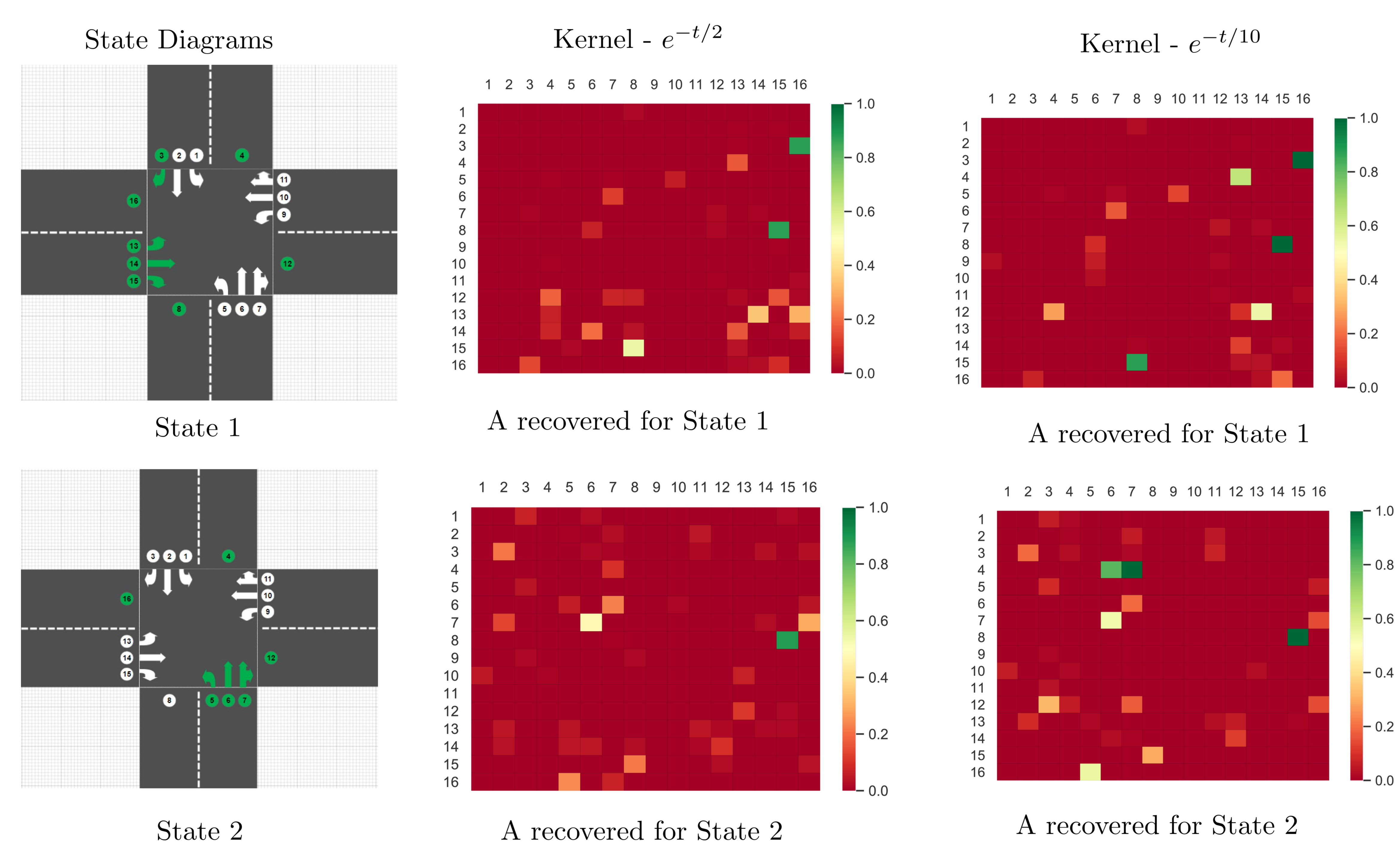
The behaviour of systems can change with time. This can be modeled as the effect of a change in the underlying parameterization which corresponds to the state of the system.

$$\lambda_{i,s}(t|\{\mathcal{H}\}) = \mu_{i,s}(t) + \sum_{j=1}^{N}\sum_{s'=1}^{M}(A_{ij})_{s,s'}\sum_{k\in K_{j,s'}}\gamma(t-t_k)$$



Real World Experimentation

Dataset consists of traffic sensor data over the span of two years from a network of 7 intersections in Montgomery County, Maryland. The data is logged as a time-stamped event every time a vehicle crosses either the Inbound Lane sensors or the Outbound Lane sensors. Other information captured with the logged event are: speed (mph), occupancy time (s), current phase (red / yellow / green), and time to end of phase(s).



- We try to model the data using two kernels, a shorter one and a longer one. The kernel lasting for a shorter period of time models strong influences only for the right lane sensors. The longer kernel picks up the interactions in all, right, through and left lanes.
- In State 1, both the kernels model the excitation between 3-16 and 8-15 sensors. The longer one does a better job for the through lane sensors 14-12 and left lane sensors 13-4.
- In State 2, the longer kernel picks up strong values for sensors 6-4 and 7-4.